On the Interpretation of Gravimetric Soil Water Retention Curve for Deformable Porous Media

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Abstract

Common approaches to obtaining material parameters from soil water retention curves (SWRCs) are discussed and the typical mistakes made in the literature are highlighted. Particular attention is given to the evaluation of the air entry value (AEV) from the gravimetric water content based SWRCs. A consistent graphical approach for the determination of AEV based on an understanding of the effects of stress history and volume change on gravimetric water retention behaviour is presented. The robustness and application of the approach are demonstrated using an extensive set of experimental data from the literature.

Keywords: Unsaturated soil, Soil Water Retention, Air-Entry Value, Deformable Soils

1. INTRODUCTION

The soil water characteristic curve (SWCC) is one of the most fundamental relationships in the mechanics of unsaturated soils that is used in the calculations relating to deformation (Matyas and Radhakrishna 1968), shear strength (Fredlund et al. 1978), effective stress (Khalili and Khabbaz 1998), multiphase flow and transport (Abriola and Pinder 1985), water retention (Brooks and Corey 1964), drainage (Hillel 1980), and appears explicitly in the constitutive formulations of pore-air and pore-water, satisfying pore-phase volume change compatibility for a change in suction (Khalili et al. 2008). Indeed, more research effort has been devoted to the quantification and characterisation of water retention behaviour in soils than any other topic in the mechanics of unsaturated porous media.

Corresponding to the suction at the transition point between saturated and unsaturated states in a SWCC plot, air entry value (AEV, shown as s_{ae} in mathematical equations) is a key unsaturated soil parameter which serves as a pivotal input in many constitutive relationships relating to the mechanics of unsaturated soils.

Numerous approaches have been proposed for the interpretation of AEV from the experimental SWCC data (Fredlund and Xing 1994; Vanapalli et al. 1998; Zhai and Rahardjo 2012). However, they have invariably been based on the assumption of soil non-deformability, neglecting the complexities arising from the volume change and stress history dependency of the water retention behaviour in unsaturated soils. In deformable materials, SWCC can assume very different shapes, even for a single soil type, depending on the volume change-suction relationship and stress history of the soil (Ng and Pang 2000; Vanapalli et al. 1999). Ignoring such dependencies can lead to gross misinterpretation of SWCC data and extraction of soil parameters that are markedly in error.

In this paper, different forms of SWCC corresponding to different soil stress histories are discussed and mathematical expressions for gravimetric water content versus suction, in both saturated and unsaturated regions, are developed for each case. Difficulties and misinterpretations frequently encountered in the literature in dealing with SWCC are examined using several numerical examples. A simple methodology is proposed for the evaluation of AEV from gravimetric water content based SWCC for deformable soils, without the need for the soil volume change data with suction. The application and appropriateness of the approach are demonstrated using an extensive array of experimental test data from the literature. It is shown that the proposed approach can be applied to a range of soil data with different initial condition, volume change behaviour and stress history.

2. COMMON APPROACHES TO DETERMINATION OF AEV FROM EXPERIMENTAL DATA

The AEV is most commonly determined graphically from the experimental SWCC data plotted in a semi-logarithmic (semi-log) plane. It is typically taken as the suction corresponding to the intersection point of two straight lines: the first line is representative of the saturated zone of SWCC, i.e. the data points at which the air phase has not invaded the pores; and the second line is represented by the desaturation zone when connected pathways of air phase are formed across the pores. The second line is usually drawn as a tangent passing through the inflection point of SWCC. For constructing the first line, depending on the data reduction approach adopted (i.e. SWCC based on degree of saturation, SWCC-S, gravimetric water content, SWCC-w, or volumetric water content, SWCC-0), a horizontal line asymptotic to SWCC-S at zero suction (e.g. Sillers et al. 2001b), or a sloping line tangent to SWCC-w and SWCC-θ at zero suction (e.g. Khalili and Khabbaz 1998; Khalili et al. 2004; Pham and Fredlund 2008) have been used. The reason behind the sloping saturated line in SWCC-w and SWCC- θ plots is that in deformable soils, prior to the air phase entering the pore space, some of the water in the pores will be expelled due to the change in the stress state of the soil. However, in SWCC-S the degree of saturation remains constant and equal to unity while the soil is saturated, and therefore the first line is drawn horizontally. Some investigators have incorrectly extended this observation to water content based soil water characteristic curves, requiring that saturated zone in SWCC-w and SWCC-θ is also represented by a horizontal line. This approach ignores the deformation of the soil with suction; i.e. during the testing, and can lead to AEVs that are several orders of magnitude in error. Examples of such deficiency have been highlighted by Zargarbashi and Khalili (2012) in relation to the work of Guan et al. (2010).

The problem of using a horizontal, instead of a sloping, line for constructing the saturated portion of SWCC-w or SWCC- θ , is in fact a special case of a more fundamental problem in evaluating AEV from SWCC-w or SWCC- θ . In many cases, the common graphical approaches for determining AEV, irrespective of using a horizontal or a sloping line; i.e. to represent the first part of SWCC-w, can lead to erroneous evaluations of AEVs for deformable soils.

Many researchers in soil science as well as geotechnical engineering consider the first break in the drying branch of SWCC-w or SWCC- θ as an indication of the point of air entry (e.g. Vanapalli et al. 1998; Fredlund and Xing 1994; Fredlund et al. 2001; Zhai and Rahardjo 2012). For a non-deformable medium, this statement is entirely correct. However, it may not be true for deformable soils depending on the mechanical properties of the soil, the value of air entry (AEV), the pore size distribution index of the soil, and the stress history of the soil.

3. EFFECT OF STRESS HISTORY ON SWCC-W FOR DEFORMABLE SOILS

To demonstrate the impact of stress history on the water content versus suction response of deformable soils, a series of numerical examples are presented for an elemental volume of a soil with mechanical properties κ , λ and p'_c ; κ is the slope of the unloading-reloading line in $v \sim \ln p'$ plane, where v is the specific volume and p' is the mean effective stress; λ is the slope of normal compression line (NCL) in the $v \sim \ln p'$ plane; and p'_c is the preconsolidation pressure of the soil. The general cases in which the water retention test is performed at a net mean stress (p_{net}) will be discussed. In this

context, the water retention response of the soil is assumed to follow Brooks and Corey model represented by:

$$S_{eff} = \left(\frac{s_{ae}}{s}\right)^{\lambda_p} \tag{1}$$

where

$$S_{eff} = \frac{S_T - S_{res}}{1 - S_{res}} \tag{2}$$

is the effective degree of saturation; S_r is the total degree of saturation; S_{res} is the residual degree of saturation; and λ_p is the pore size distribution index of the soil.

Three stress states: i) normally consolidated, ii) overconsolidated with $p'_c < p_{net} + s_{ae}$ and, iii) overconsolidated with $p'_c > p_{net} + s_{ae}$ are considered, reflective of the various stress histories of a soil. Effective stress is used for the quantification of the volume change of soil due to a change in suction, expressed as (Bishop 1959):

$$\sigma' = (\sigma - u_a) + \chi (u_a - u_w) \tag{3}$$

in which σ' is the effective stress; σ is the total stress; u_a and u_w are the pore air and pore water pressures, respectively; and $\sigma - u_a$ is the net stress, σ_{net} . $u_a - u_w = s$ is the matric suction, and χ is the effective stress parameter, determined in this work using the relationship proposed by Khalili and Khabbaz (1998).

3.1. Normally consolidated soil or soil prepared from slurry

Soils with the externally applied stress equal to the preconsolidation pressure, p'_c , (but less than the AEV) or soils prepared from a slurry, behave as normally consolidated (NC) in the saturated zone of the SWCC ($s \le s_{ac}$). In such soils, the initial part of SWCC-w curve, in a semi-log plane, can be approximated with a single line having a slope corresponding to the slope of the normal compression line, λ , as the volume of water lost during this stage is equal to the decrease in the volume of the voids. After the start of desaturation, the change of water content with suction in the SWCC-w plot is controlled primarily by the pore size distribution of the soil. The suction at the intersection of these two parts of SWCC-w is regarded as AEV.

In this case, the total form of SWCC-*w* is derived as:

$$w = \begin{cases} w_0 - \frac{2.3\lambda}{G_s} \log\left(\frac{s + p_{net}}{s_0 + p_{net}}\right) & ; s < s_{ae} \quad (4a) \\ \\ w_{ae} \left((1 - S_{res}) \left(\frac{s_{ae}}{s}\right)^{\lambda_p} + S_{res}\right) & ; s > s_{ae} \quad (4b) \end{cases}$$

In Equation (4), G_s is the specific gravity of solid grains, and w_{ae} is the gravimetric water content at AEV, calculated as

$$w_{ae} = w_0 - \frac{2.3\lambda}{G_s} \log\left(\frac{s_{ae} + p_{net}}{s_0 + p_{net}}\right) \tag{4c}$$

where w_0 is the reference water content at the reference reference stress state, $s_0 + p_{net}$. Also note that since Equation (4) is derived for normally consolidated soils, hence $p_{net} = p'_c$ in this equation. A graphical representation of Equation (4), in terms of water content versus $s + p_{net}$, for the case of $p_{net} = p'_c \neq 0$, is given in Figure 1. The SWCC contains two separate parts in this case, a linear part where the soil remains saturated and behaves as a normally consolidated soil followed by a non-linear part where the soil is desaturated. Note that in the cases where $p_{net} = p'_c \neq 0$, the saturated part of the SWCC-w will appear as a line only when the data is plotted versus $s + p_{net}$ instead of pure suction as shown in Figure 1. It is evident from Figure 1 that using the common procedure of drawing a horizontal line or a straight line through the initial data points, and a best fit through the rest of the data to determine AEV (shown by long and short dashed lines) can lead to gross miscalculations of AEV. The true AEV is the value of suction corresponding to the intersection of the two straight lines representing the steeply inclined saturated section of the response and the tangent to the initial portion of the desaturation response. The task of correctly identifying AEV becomes further exacerbated when the slopes of the lines before and after AEV are of similar magnitude as depicted in Figure 1b.



Figure 1. Schematic SWCCs for normally consolidated soils or soils with $p_{net} = p'_c$ presented in terms of water content with possible miscalculated AEVs, with (a) different, and (b) similar, slopes before and after the point of air entry

3.2. Overconsolidated soil $(p'_c < p_{net} + s_{ae})$

For the case of overconsolidated soils where $p'_c < p_{net} + s_{ae}$, the total form of SWCC-*w* in a semilog plane can in turn be derived as follows:

For
$$s + p_{net} < p_c < s_{ae} + p_{net}$$

$$w = w_0 - \frac{2.3\kappa}{G_s} \log\left(\frac{s + p_{net}}{s_0 + p_{net}}\right);$$
(5a)

for
$$p'_c < s + p_{net} < s_{ae} + p_{net}$$
:

$$w = w_0 - \frac{2.3\kappa}{G_s} \log\left(\frac{p'_c}{s_0 + p_{net}}\right) - \frac{2.3\lambda}{G_s} \log\left(\frac{s + p_{net}}{p'_c}\right)$$
(5b)

and for $p'_c < s_{ae} + p_{net} < s + p_{net}$:

$$w = w_0 - \frac{2.3\kappa}{G_s} \log\left(\frac{p'_c}{s_0 + p_{net}}\right) - \frac{2.3\lambda}{G_s} \log\left(\frac{s + p_{net}}{p'_c}\right)$$
(5c)

in which

$$w_{ae} = w_0 - \frac{2.3\kappa}{G_s} \log\left(\frac{p'_c}{s_0 + p_{net}}\right) - \frac{2.3\lambda}{G_s} \log\left(\frac{s_{ae} + p_{net}}{p'_c}\right)$$
(5d)

The graphical representation of Equations (5), in terms of water content versus $s + p_{net}$, is given in Figure 2. Again, it is evident that using the usual approaches to determination of AEV from SWCC-w may lead to gross miscalculations. Of particular interest in this case is that the preconsolidation pressure, p'_c , is directly reflected in SWCC-w as a break point at the value of suction equal to $p'_c - p_{net}$ (first break point in Figure 2). At this break point, the soil is still saturated, and the stress state of the soil has merely changed from overconsolidated to normally consolidated. In the literature, this break point is often taken, by mistake, as the point of air entry for the soil.



Figure 2. Schematic SWCCs for overconsolidated soils with $p'_{c} < p_{net} + s_{ae}$ presented in terms of water content with possible miscalculated AEVs, with (a) different, and (b) similar, slopes before and after the point of air entry

3.3. Overconsolidated soil $(p'_c > p_{net} + s_{ae})$

For the case of overconsolidated soils where $p'_c > p_{net} + s_{ae}$, the total form of SWCC-*w* in semi-log plane can be derived as:

$$w = \begin{cases} w_0 - \frac{2.3\kappa}{G_s} \log\left(\frac{s + p_{net}}{s_0 + p_{net}}\right) & ; s < s_{ae} \quad (6a) \\ \\ w_{ae} \left((1 - S_{res})\left(\frac{S_{ae}}{s}\right)^{\lambda_p} + S_{res}\right) ; s > s_{ae} \quad (6b) \end{cases}$$

where

$$w_{ae} = w_0 - \frac{2.3\lambda}{G_s} \log \left(\frac{s_{ae} + p_{net}}{s_0 + p_{net}} \right) \tag{6c}$$

A schematic representation of SWCC-w for overconsolidated soils with $p'_c > p_{net} + s_{ae}$, is presented in Figure 3. As shown, the evaluation of AEV is less prone to error in this case compared to normally consolidated and/or overconsolidated soils with $p'_c < p_{net} + s_{ae}$.

For overconsolidated soils with $p'_c > p_{net} + s_{ae}$, the soil volume change during application of suction is small, and the preconsolidation pressure of the soil does not feature in SWCC-*w* response of the soil. More precisely, SWCC-*w* will only show a single break point, corresponding to the point of air entry. In this case, the slope of the saturated portion of the curve is dependent upon the slope of unloading-reloading curve in $\nu \sim lnp'$ plane, κ , and the common approach of defining AEV at the intersection of the two straight portions of the water retention response will yield an accurate estimation of AEV. Note that in the general case of non-zero net mean stress, in order to calculate the AEV the value of the horizontal axis at the point of air entry should be reduced by p_{net} .



Figure 3. Schematic SWCC for overconsolidated soils with $p'_{c} > p_{net} + s_{ae}$ presented in terms of water content

4. PROPOSED METHOD FOR DETERMINATION OF AEV FROM SWCC-W

From the examples provided, it is evident that the suction corresponding to AEV may not always coincide with a clear break point in a SWCC-*w* plotted in a semi-log plane. In fact, in some cases, more than one break may also be identified in the SWCC-*w* plot for a soil. These cases have been proven to be difficult to handle in the literature as the existing approaches for determination of AEV from SWCC data can lead to erroneous estimations.

In order to minimize error in the determination of AEV, particularly for the cases highlighted above, it is suggested that the gravimetric water content data are plotted versus $s + p_{net}$ (suction plus net mean stress) on the semi-log scale and versus s (suction) on the log-log scale, in the same graph. A secondary vertical axis within the same range of values used for the primary axis may be adopted for this purpose; e.g. see Figure 4 for the previously discussed case of overconsolidated soils with $p'_c < p_{net} + s_{ae}$. As shown, plotting data in the two mentioned scales has the following advantages: 1) According to the existing SWCC models and based on numerous experimental data, by plotting SWCC in a log-log scale, the unsaturated part of the response may be represented by a straight line, with a slope related to the pore size distribution index of the soil, λ_p . Therefore, in the log-log scale

the unsaturated part of the response can be readily distinguished by a linear behaviour. On the other hand, in the semi-log plane it is the saturated part of the curve that is identified by a linear or bilinear behaviour depending on the soil stress history. Hence, the two curves are complementary, and when drawn together on a single graph results in an easier identification of AEV. In this approach, the AEV is identified as the point where the saturated data deviate from the straight line drawn to the unsaturated part of the plot on the log-log scale. Also on the semi-log scale, the unsaturated data deviate from the line drawn through the saturated part of the plot at the air entry point where the AEV can be calculated by subtracting p_{net} from the value of the horizontal coordinate at the separation point on the semi-log plot. The two curves should yield the same values air entry suction which can be regarded as the AEV of the soil. The slopes of the above-mentioned linear parts also have clear physical meanings, and can be utilised to calculate soil parameters for the purposes of constitutive modelling; 2) Drawing two plots with two different scales in the same graph also minimizes the possibility of missing or misrepresenting the location of the point of air entry when the slopes of the SWCC-w graph before and after AEV are similar (see Figure 4). Even if the break may not be detectable on one or both of the plots, it will be possible to determine the AEV by locating the suction value at which data depart from linear/bilinear responses on both plots.

For the case of overconsolidated soils with $p'_c < p_{net} + s_{ae}$ (Figure 4), the first break in the SWCC-*w* plot corresponds to the preconsolidation pressure of the soil and must not be taken as AEV. For these soils, the air entry will occur at a greater suction which may or may not appear as a distinct break in the SWCC-*w* graph, plotted on a semi-log plane. As such, in an overconsolidated soil, if two break points appear in the SWCC-*w* graph, the first point is likely to be due to the preconsolidation pressure of the soil, and only the second break point may be related to AEV. The most difficult SWCC-*w* graphs to interpret are those in which the soil is overconsolidated but it contains only a single break point as shown in Figure 4. In such cases, without a knowledge of the preconsolidation pressure of the soil, or adopting the two scale plotting approach proposed, it will be very difficult if not impossible to identify the point of air entry and in all likelihood the sudden change in the slope of SWCC-*w* will be taken as AEV, which may not be correct. Despite this, none of the existing mathematical expressions presented in the literature for modelling SWCC-*w* as AEV, irrespective of the stress history of the soil (e.g. Fredlund and Xing 1994; Pham and Fredlund 2008; van Genuchten 1980).

Another advantage of the method proposed is that it essentially eliminates the need for measuring the volume change of the sample during SWCC test. Correct estimation of AEV is possible by simply plotting SWCC-*w* on two semi-log and log-log scales and identifying the linear parts of each plot. Provided due attention is given to the possibility of having a bilinear response in the initial part of the semi-log graph, AEV can be estimated with limited or no data on the stress history of the sample or volume change of the sample during the test. Such estimation of AEV will not be possible using the common semi-log plots of SWCC-*w* data. At this point it is important to note that AEV is not a unique property of soil and it varies with volume change, stress history, the way a sample is prepared, etc. Therefore, discussions in this paper pertain only to the determination of AEV for a given soil with a given stress history, void ratio, structure and fabric.



Figure 4. Schematic SWCC for overconsolidated soils with $p'_c < p_{net} + s_{ae}$ presented in terms of gravimetric water content in both semi-log and log-log scales (for $S_{res} = 0$) along with the proposed method to determine AEV

5. EXPERIMENTAL VALIDATION

In this section, the application of the proposed method to determine AEV and other soil parameters from SWCC data is investigated using the experimental data for three different categories of soils discussed in previous sections.

The first data taken from Fredlund and Houston (2013), pertains to a SWCC test on the highly plastic Regina Clay prepared from slurry and preconsolidated under a very small load prior to SWCC testing which falls in the category of normally consolidated soils as described previously. The test was performed on drying path of SWCC at zero net mean stress ($p_{net} = 0$).

The experimental SWCC-w data are presented in Figure 5, plotted both in semi-log and log-log planes. Two linear parts in the curves are identified: one representing the saturated region on the semi-log plot, and the other representing the unsaturated region on the log-log plot. The point of departure from linear behaviour in each of the plots is identified as the AEV of the soil with $s_{ae} = 4000$ kPa. The slopes of the two straight lines in Figure 5a can also be used to calculate: $\lambda = 0.249$, and $\lambda_p = 0.43$. Figure 5b confirms that the estimated AEV agrees very well with AEV obtained from SWCC-S data. Also notice that the form of SWCC-S remains unaffected by the stress history as the influence of volume change during testing is included in the data reduction of the water retention test results. This again confirms that when volume change data during testing is available, the most appropriate form of presentation of water retention results is SWCC-S.

The second data is related to a SWCC test on a soil mixture of 20% pure Speswhite kaolin, 10% London clay and 70% HPF4 silica silt reported by Cunningham et al. (2003). The reconstituted sample was prepared in oedometer to a vertical effective stress of 200 kPa before the SWCC test and hence it falls in the category of overconsolidated soils. Finally, the third test have been performed by the authors on a mixture of 65% Sydney sand and 35% Kaolin clay. This sample was prepared from slurry and was preconsolidated to a high vertical stress of 400 kPa in oedometer prior to the water retention test. The test was performed on drying path at zero net mean stress ($p_{net} = 0$).



Figure 5. Drying path SWCC of normally consolidated Regina clay at zero net mean stress: (a) Experimental data in SWCC-*w* space along with the proposed graphical method to obtain parameters and (b) Experimental data in SWCC-S space along with the predicted curve using the parameters from the graphical method (Data after Fredlund and Houston 2013)



(a)

(b)

Figure 6. Drying path SWCC of a clay-silt mixture at zero net mean stress: (a) Experimental data in SWCC-*w* space along with the proposed graphical method to obtain parameters and (b) Experimental data in SWCC-**S** space along with the predicted curve using the parameters from the graphical method (Data after Cunningham et al. 2003)

Figure 6a presents the drying path SWCC-w of the soil using both semi-log and log-log scales according to the proposed method. Two pronounced breaks are identified in the semi-log plot. The first one, at a lower suction, is due to the preconsolidation of the soil, and the second one, at a higher suction, is attributed to the AEV, which is estimated as 500 kPa. Using the proposed graphical technique, a bilinear curve is fitted to the experimental points prior to AEV on the semi-log plot in Figure 6a, representing the soil behaviour in the saturated zone. Another line is drawn through the experimental data on the log-log plot, representing the response in the unsaturated zone. Once again, the point of deviation from the linear behaviour in both plots is defined as the air entry point (Figure 6a). Other model parameters obtained from the data are: $\kappa = 0.007$; $\lambda = 0.07$; and $\lambda_p = 0.69$. It is noted that the first break occurs at a suction value of 150 kPa, which is slightly lower than the vertical preconsolidation pressure of the sample (i.e. 200 kPa). This is due to the difference between the isotropic preconsolidation pressure, captured during suction loading, and that corresponding to

oedometric consolidation obtained from the one-dimensional loading of the sample. Similar to the previous examples, the same AEV and λ_{p} are obtained from SWCC-S data as shown in Figure 6b.

The third example is related to Thu et al. (2007) data on coarse kaolin classified as high plasticity silt (MH). The sample was compacted at maximum dry density and optimum water content and then subjected to a net confining stress of 100 kPa. The SWCC was performed on the sample without removing the stress.

The experimental SWCC-w for this sample is plotted in Figure 7a in both log-log and semi-log scales according to the proposed method. A single break point is observed in the semi-log and log-log SWCC-w plots with the corresponding suction value less than 100 kPa from both curves. The slope of the linear part before this break point in the semi-log plot is very low and around the re-compression index, κ , of reported for the soil. This soil falls under the category of overconsolidated soils with $p'_c > p_{net} + s_{ae}$ which is due mainly to the sample preparation method described above. The proposed method yields an AEV of 60 kPa, and also a λ_p of 0.74 which is consistent with the experimental SWCC-S as shown in Figure 7b.



Figure 7. Drying path SWCC of a compacted silt at 100 kPa net mean stress: (a) Experimental data in SWCCw space along with the proposed graphical method to obtain parameters and (b) Experimental data in SWCC-S space along with the predicted curve using the parameters from the graphical method (Data after Thu et al. 2007)

6. CONCLUSIONS

Pitfalls in the interpretation of water content based SWCC and evaluation of AEV are discussed. Different forms of SWCC-*w* corresponding to different stress histories are examined. It is shown that extreme caution should be exercised in the interpretation of water retention data presented in terms of gravimetric water content versus suction. Specifically, it has been shown that methodologies which do not take into account the stress history of soil may yield grossly erroneous estimations of AEV. Where volume change data are measured; i.e. during the water retention test, the SWCC should be presented in terms of degree of saturation versus suction for the correct estimation of AEV. For cases in which volume change is not measured, a simple graphical technique is proposed for the evaluation of AEV from the SWCC data presented in terms of gravimetric water content. The validity and application of the model are demonstrated using experimental data from the literature.

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