Modelling of Dynamic Pile-Soil-Pile Interaction Using Radiation Elements

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Abstract

In this paper, a new type of elements called radiation discs is introduced to model dynamic pile-soil-pile interaction and to deal with the radiation conditions at infinite domains. The pile group system can be modelled using beam-column elements, while the radiation discs are defined at the nodal points of the elements to model the pile-soil-pile interaction. A Boussinesq-type loading distribution is proposed to act on the discs to achieve the proper mode of deformation at the cross sections of piles. Using radiation discs, the discretisation of the domain is only required along the length of piles, while the discretisation of soil medium, top free surface boundary, and cross sections of piles are avoided. Numerical examples are presented to demonstrate the application of the method and to investigate the influence of excitation frequency and pile spacing on dynamic response of pile groups.

Keywords: Soil-pile interaction, Pile group, Dynamic, Radiation elements, Boussinesq distribution

1. INTRODUCTION

The description of pile-soil-pile interaction under dynamic loading conditions is a key area of research in modern geotechnical engineering. This is due to the wide applications of pile groups in structures exposed to dynamic loads, including bridges, machinery foundations, and offshore platforms. A variety of numerical and analytical techniques has been developed to investigate the dynamics of piles and pile groups. Among the notable contributions have included the works of Wolf and Von Arx (1978), Nogami (1979), Poulos and Davis (1980), Kaynia (1982), Sheta and Novak (1982), Senm *et al.* (1985); Pak and Jennings (1987), Rajapakse and Shah (1989); Mamoon *et al.* (1990); Gazetas *et al.* (1991), EI-Marsafawi *et al.* (1992), Liu and Novak (1994), Randolph (2003), Barros (2003), Noorzad *et al.* (2005), Cairo *et al.* (2005), Padrón *et al.* (2007), Shahmohamadi *et al.* (2011), Shahmohamadi *et al.* (2013), and Gharahi *et al.* (2014). Most of the methods used for a direct and complete analysis of pile groups involve either the discretisation of the whole domain, e.g. Finite Element and Finite Difference Methods, or the boundary of the domain including top free surface and pile-soil interfaces, i.e. Boundary Element Method. These numerical methods generally requires significant computational effort for three-dimensional analysis of pile groups under dynamic loads, in particular when the pile group consists of a large number of piles.

This paper presents a novel numerical method for dynamics of piles and pile groups. The theoretical approach adopted is an extension of the method proposed by Muki and Sternberg (1969, 1970) to include the inertia as well as anisotropy effects. In the hybrid method, the piles are modelled using finite elements, while massless rigid radiation elements are defined at the nodal points of the elements to model the wave propagation in the surrounding medium. The elastodynamic response of the radiation elements buried at different depths in an anisotropic half-space is derived in a transform domain using a set of complete potential functions proposed by Noorzad *et al.* (2003) and Eskandari-Ghadi (2005). A Boussinesq-type distribution is defined to act on the radiation elements to attain the proper mode of deformation along the cross-sections of piles in the soil medium system. In the method, the discretisation is only required along the length of piles, while discretisation of the surrounding medium,

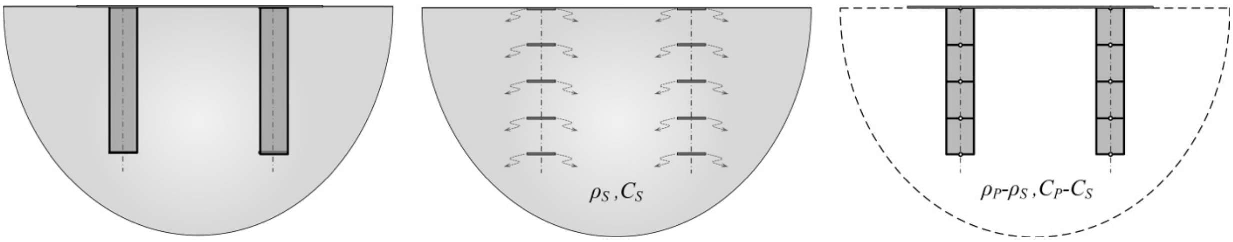

Figure 1. Modelling of dynamic pile-soil-pile interaction: soil-pile group system (left), radiation elements (middle), finite elements (right).

top free surface boundary, and along the cross sections of piles are avoided. Numerical examples are presented to demonstrate the application of the method and to investigate the influence of excitation frequency and pile spacing on dynamic response of pile groups.

2. MODELLING OF DYNAMIC PILE-SOIL-PILE INTERACTION

In this numerical method, the pile group system is modelled using beam-column elements, while massless rigid radiation discs are defined at the nodal points of the elements to model the pile-soil-pile interaction, see Figure 1. A Boussinesq-type distribution is defined to act on the radiation discs to attain the rigid mode of deformation along the cross-sections of piles. In a cylindrical coordinate system (r,θ,z), this distribution can be expressed as

$$f_{Pi}^{f}(r,\theta,z,t)=\begin{cases}\dfrac{F_i(\theta,z,t)}{2\pi a^2\sqrt{1-\dfrac{r^2}{a^2}}} & , \quad r<a\\ 0 & , \quad r\geq a\end{cases} \tag{1}$$

where $F_i(\theta,z,t)$ depends on direction and magnitude of external forces and a is the radius of the radiation disc. By using this equation, the discretisation along the cross sections of piles is avoided in the proposed scheme. The compatibility condition between the two systems in z-direction is satisfied by considering the same displacement field for the nodal points of the pile elements and the radiation discs. Hence, the dynamic stiffness matrix of the pile-soil-pile system $[K_T]$ is derived from

$$[K_T]=[K_S]+[K_P] \tag{2}$$

where $[K_P]$ and $[K_S]$ are the piles and the radiation discs stiffness matrices, respectively. The dynamic stiffness matrix of the radiation discs is obtained by inverting the flexibility matrix. This matrix is derived through applying unit dynamic load at each radiation disc, and determining the response of all the discs in the group. The method is general and can readily be extended to simulate any arbitrary pile group configurations and pile-soil-pile interaction problems with more complex loading and boundary conditions.

3. DYNAMIC RESPONSE OF RADIATION ELEMENTS

The elastodynamic response of the radiation discs buried at different depths in a transversely isotropic half-space is derived in a transform domain using a set of complete potential functions (Noorzad *et al.*, 2003; Eskandari-Ghadi, 2005). For a transversely isotropic half-space, the displacement functions in the frequency domain can be expressed in terms of two scalar potential functions X and Ψ as

$$u = -\alpha_3 \frac{\partial^2 \Psi}{\partial r \partial z} - \frac{1}{r}\frac{\partial X}{\partial \theta}$$
$$v = -\frac{\alpha_3}{r}\frac{\partial^2 \Psi}{\partial \theta \partial z} + \frac{\partial X}{\partial r} \tag{3}$$
$$w = (1+\alpha_1)(\nabla^2_{r\theta} + \beta \frac{\partial^2}{\partial z^2} + \frac{\rho_0 \omega^2}{1+\alpha_1})\Psi$$

where u, v, and w are displacement components in r, θ, and z directions in the cylindrical coordinate system, respectively, $A_{11} = E_S\left(1-\left(E_S / E'_S\right)v'^2_S\right)/\left(\left(1+v_S\right)\Delta\right)$, $A_{12} = A_{11} - 2A_{66}$, $A_{13} = E_S v'_S / \Delta$, $A_{33} = E'_S\left(1-v_S\right)/\Delta$, $A_{66} = E_S / 2\left(1+v_S\right) = G_S$, $\alpha_1 = \left(A_{11}+A_{12}\right)/\left(A_{11}-A_{12}\right)$, $A_{44} = G'_S$, $\alpha_2 = A_{44} / A_{66}$, $\alpha_3 = \left(A_{13}+A_{44}\right)/A_{66}$, $\nabla^2_{r\theta} = \partial^2 / \partial r^2 + 1/\left(r\left(\partial / \partial r\right)\right) + 1/\left(r^2\left(\partial^2 / \partial \theta^2\right)\right)$, $\beta = \alpha_2 / \left(1+\alpha_1\right)$, $\rho_0 = \rho_S / A_{66}$, $\Delta = 1 - v_S - 2E_S v'^2_S / E'_S$, ω is the circular frequency, E_S and E'_S are Young's moduli in the plane of isotropy and in the direction normal to it, v_S and v'_S are in-plane and normal Poisson's ratios characterising normal strains in the plane of isotropy, G_S and G'_S are in-plane and normal shear moduli, respectively, and ρ_S is the soil density.

Using the potential functions, the wave equations for the transversely isotropic medium can be expressed as

$$\nabla^2_0 X = 0$$
$$\nabla^2_1 \nabla^2_2 \Psi + B\rho_S\, \omega^2 \frac{\partial^2 \Psi}{\partial z^2} = 0 \tag{4}$$

where $\nabla^2_i = \nabla^2_{r\theta} + 1/\left(s_i^2\left(\partial^2 / \partial z^2\right)\right) + \rho_0 \omega^2 / \mu_i, \left(i = 0,1,2\right)$, $\mu_0 = 1$, $\mu_1 = \alpha_2$, $\mu_2 = 1+\alpha_1$, $B = \left(1+A_{33} / A_{44}\right)/A_{11} - \left(1/\mu_1 s_2^2 + 1/\mu_2 s_1^2\right)/A_{66}$, $s_0^2 = 1/\alpha_2$, and s_1 and s_2 are the roots of $A_{33}A_{44}s^4 + \left(A_{13}{}^2 + 2A_{13}A_{44} - A_{11}A_{33}\right)s^2 + A_{11}A_{44} = 0$ (Lekhnitskii,1981).

The displacement and stress boundary conditions can also be expressed in terms Potential functions (Shahbodagh, 2008; Shahbodagh *et al.*, 2017). Using the Fourier decomposition with respect to the angular coordinate and the Hankel transform with respect to the radial coordinate, the Fourier components of the displacement vector can be expressed as

$$w_m = \left(1+\alpha_1\right)\int_0^\infty \xi \left(-\xi^2 + \beta \frac{d^2}{dz^2} + \frac{\rho_0 \omega^2}{\mu_2}\right) \Psi^m_m J_m(\xi r) d\xi \tag{5}$$

$$u_m = \frac{\alpha_3}{2}\int_0^\infty \left(J_{m+1}(\xi r) - J_{m-1}(\xi r)\right)\xi^2 \frac{d\Psi^m_m}{dz} d\xi - \frac{i}{2}\int_0^\infty \left(J_{m+1}(\xi r) + J_{m-1}(\xi r)\right)\xi^2 X^m_m d\xi \tag{6}$$

$$v_m = -\frac{i\alpha_3}{2}\int_0^\infty \left(J_{m+1}(\xi r) + J_{m-1}(\xi r)\right)\xi^2 \frac{d\Psi^m_m}{dz} d\xi - \frac{1}{2}\int_0^\infty \left(J_{m+1}(\xi r) - J_{m-1}(\xi r)\right)\xi^2 X^m_m d\xi \tag{7}$$

where X_m^m and Ψ_m^m are the mth-order Hankel transform of the mth Fourier coefficients of X and Ψ, respectively, and J_m is the Bessel function of the first kind of order m.

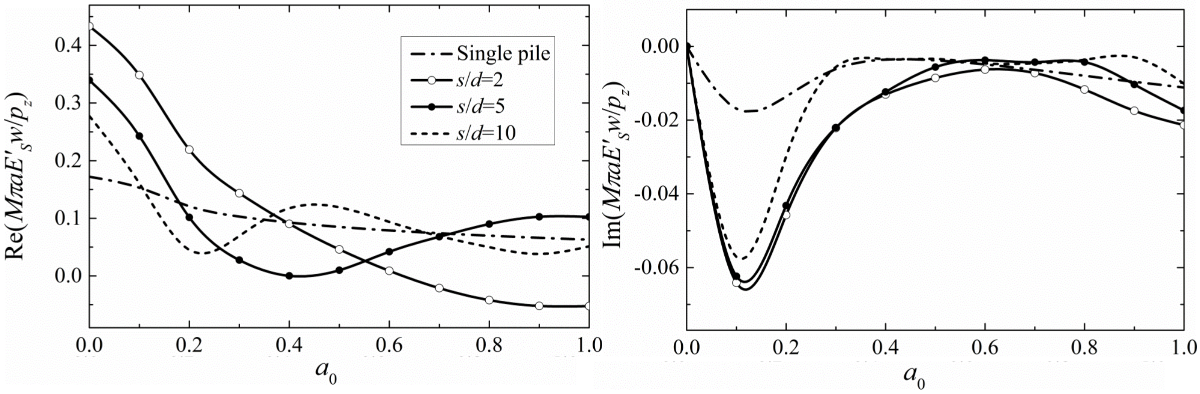

Figure 2. Influence of pile spacing and excitation frequency on dynamic response of 2×2 pile groups: Normalized vertical compliance vs. normalized frequency. (*M*: Number of piles)

The Boussinesq-type distribution specified in Equation (1) is considered to act on the radiation discs. In vertical and horizontal loading conditions, this distribution with unit magnitude can, respectively, be expressed as

$$f(r,\theta)=\begin{cases}1/\left(2\pi a^2\sqrt{\dfrac{1-r^2}{a^2}}\right), & r<a\\ 0, & r\geq a\end{cases},\quad g_r(r,\theta)=0\,,\quad g_\theta(r,\theta)=0 \tag{8}$$

$$f(r,\theta)=0\,,\quad \{g_r,g_\theta\}(r,\theta)=\begin{cases}\{\cos\theta,-\sin\theta\}/\left\{2\pi a^2\sqrt{\dfrac{1-r^2}{a^2}}\right\}, & r<a\\ 0, & r\geq a\end{cases} \tag{9}$$

Using Equations (5)-(9), the dynamic flexibility matrix of the radiation discs can be obtained for different modes of vibration. The dynamic stiffness matrix of the radiation discs defined in Equation (2) is derived by inverting the radiation discs flexibility matrix. Similar to finite element analysis, any displacement constraints due to pile cap can be applied to the nodal points of the pile elements and radiation discs.

4. NUMERICAL RESULTS

In this section, the dynamic response of 2×2 pile groups with different spacing ratios, i.e. s/d=2, 5, 10, embedded in transversely isotropic half-space is presented. The pile slenderness ratio $l/d=15$, the density ratio $\rho_P/\rho_S=1.5$, and the pile stiffness ratio $E_P/E'_S=1000$ are assumed in the analysis. The material properties adopted for the half-space are E_S/G'_S=5.0, E'_S/G'_S=2.5, and $\nu_S=\nu'_S$=0.25. Figure 2 shows the normalized vertical compliance of the groups versus the dimensionless frequency $a_0=\omega d\sqrt{\rho_S/A_{44}}$. For benchmarking, the figures also include the results from a single-pile response. It is observed that the response of the pile group is markedly affected by the interaction among the piles and strongly depends on the excitation frequency and the pile spacing ratio. The analysis clearly shows that the frequency dependent response of the group cannot be deduced from the study of the behaviour of single piles, and a complete treatment of the interaction problem is crucial in pile group analysis. However, as the pile spacing increases, the dynamic response of the group approaches the single-pile response.

5. CONCLUSIONS

A new type of elements called radiation discs is introduced to model dynamic pile-soil-pile interaction. The pile group system is modelled using beam-column elements, while the radiation elements are defined at the nodal points of the elements to model the wave propagation through the medium. A Boussinesq-type loading distribution is proposed to act on the radiation elements to achieve the proper mode of deformation at the cross sections of piles. Numerical examples are presented and the effects of excitation frequency and pile spacing on dynamic compliance of pile groups are investigated. The analysis clearly shows that the frequency dependent response of the pile group cannot be deduced from the study of the behaviour of single piles, and a complete treatment of the interaction problem is crucial in pile group analysis.

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