A thermodynamically consistent model on coupled creep and damage behaviour of porous unsaturated rocks

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ABSTRACT
The time-dependent response of rocks is strongly influenced by the presence of pore fluids, particularly for unsaturated materials. This paper presents a theoretical framework for modelling the coupled creep and damage behaviour of unsaturated rocks based on thermodynamic principles. The coupled phenomena of delayed inelastic strains and microcracks are modelled using the theories of viscoplasticity and continuum damage mechanics, both governed by a plastic effective stress. This framework strikes a balance between different requirements in constitutive modelling: capability to reproduce experimental observations, simplicity for practical use and thermodynamic consistency. The relevance of the model is illustrated by confronting its predictions to experimental data.

1. INTRODUCTION

Quantifying the time-dependent deformation of rocks is crucial for many engineering applications, such as mining excavation, oil and gas extraction or geological disposal facilities (Arson, et al., 2012; Bui, et al., 2014). The delayed response of rocks may come from different mechanisms such as crystal dislocation, diffusion creep and microcracking (Boukharov, et al., 1995). Compared to the first two mechanisms, the latter often implies a larger volume expansion and consequently, failure of the rock sample. Moreover, the time-dependent deformation can be significantly affected by the presence of pore fluids (Grgic & Amitrano, 2009).

In the literature, very few constitutive models consider the long-term behaviour of unsaturated rocks (Hoxha, et al., 2005; De Gennaro & Pereira, 2013; Zhang, et al., 2013). These models require a quite large number of parameters and, last but not least, their thermodynamic consistency has hardly ever been discussed. In this context, this paper presents a thermodynamically rigorous modelling framework in order to describe the time-dependent behaviour of unsaturated rocks, while maintaining a relative simplicity in order to remain applicable. This framework relies on the theories of poromechanics (Coussy, 2004) and irreversible thermodynamics, including specifically adapted creep damage and viscoplastic mechanisms.
2. PROPOSED MODELLING FRAMEWORK

In the following, the continuum mechanics sign convention is adopted (tensile stress and strain positive); scalars are denoted in normal character while vectors and tensors are in bold. Isothermal condition is assumed; strains, porosity changes and displacements are infinitesimal. Rock is assumed to be mechanically isotropic and saturated by a liquid (with a pressure $p_w$) and a gas (with a pressure $p_g$). For simplicity, only isotropic damage, represented by a scalar internal variable $D$, and isotropic hardening, represented by a scalar variable $\gamma_{vp}$, are considered. The total strain tensor $\varepsilon$ and total porosity variation $\Delta \phi$ are supposed to be decomposable into elastic and viscoplastic parts, denoted by the superscripts $e$ and $vp$, respectively:

$$\varepsilon = \varepsilon^e + \varepsilon^{vp}; \quad \Delta \phi = \phi - \phi_0 = \phi^e + \phi^{vp}$$

where $\phi_0$ is the initial porosity.

Denoting the average pore pressure $p^* = p_w S_w + p_g S_g$ and capillary pressure $p_c = p_g - p_w$, with $S_w$ and $S_g$ respectively the degrees of saturation of the liquid and gaseous phases, the inequality of Clausius-Duhem reads (Coussy, 2004):

$$\Phi = \sigma : \dot{\varepsilon} + p^* \dot{\phi} - \phi p_c S_w - \dot{\Psi}_s \geq 0$$

where $\Phi$ is the dissipation related to the solid skeleton, $\sigma$ is the stress tensor, the over-dot notation denotes a time derivative and $\Psi_s = \Psi_s(\varepsilon^e, \phi^e, D, \gamma_{vp}) + \Phi U(\phi, S_w)$ is the total free energy of the skeleton per unit overall volume; $\Psi_s$ denotes the free energy of solid matrix and $U$ the free energy of the interfaces per unit volume of porous space (Coussy, 2004).

Defining the potential $G_s = \Psi_s - \pi_{eq} \phi^e$, the inequality (2) can be rewritten as follows:

$$\Phi = \sigma : \dot{\varepsilon}^e - \phi^e \pi_{eq} - \left(\phi p_c + \frac{\partial (\Phi U)}{\partial S_w}\right) S_w + \sigma : \dot{\varepsilon}^{vp} + \pi_{eq} \dot{\phi}^{vp} - \dot{\phi}_s \geq 0$$

where the (thermodynamically) equivalent pore pressure $\pi_{eq}$ is defined by: $\pi_{eq} = p^* - \frac{\partial (\Phi U)}{\partial \phi}$.

Invoking other results using a micromechanical approach, Coussy and Dangla (Coussy, 2004) proved that this equivalent pressure should take the following form:

$$\pi_{eq} = p^* - \frac{2}{3} U$$

The two last definitions of $\pi_{eq}$ lead to $U(\phi, S_w) = (\phi/\phi_r)^{-1/3} U(\phi_r, S_w)$, where $\phi_r$ is a reference porosity. In rocks, the variation of the porosity is generally very small (Zhang, et al., 2010) thus the term $(\phi/\phi_r)^{-1/3}$ is close to unity and $U$ can be assumed to be independent of $\phi$.

For purely reversible processes, $\dot{\varepsilon}^{vp} = \dot{\phi}^{vp} = \dot{D} = \gamma_{vp} = \dot{\phi} = 0$, (3) allows to deduce the following state laws:

$$\sigma = \frac{\partial G_s}{\partial \varepsilon^e}; \quad \phi^e = - \frac{\partial G_s}{\partial p_w}$$
\[ p_c = -\frac{\partial U}{\partial S_w} \]  

(6)

in which Equation (6) describes the water retention relation, assumed reversible here to simplify, which leads to the determination of the interfacial energy: 

\[ U = \int_{S_w}^1 p_c(\dot{S}_w) d\dot{S}_w. \]

Assuming that (5) and (6) hold even during irreversible processes, (3) becomes:

\[ \Phi = \sigma : \dot{e}^{vp} + \pi_{eq} \dot{\phi}^{vp} + Y_d \dot{D} - \alpha_{vp} \dot{\gamma}_{vp} \geq 0 \]  

(7)

where \( Y_d \) and \( \alpha_{vp} \) are the thermodynamic forces conjugate to damage and viscoplastic hardening, defined by:

\[ Y_d = -\frac{\partial G_s}{\partial D} \]  

(8)

\[ \alpha_{vp} = \frac{\partial G_s}{\partial Y_{vp}} \]  

(9)

Since evaluating the viscoplastic porosity change is generally very difficult, the following assumption proposed by (Coussy, 2004) is adopted:

\[ \dot{\phi}^{vp} = b^{vp} \epsilon^{vp} \]  

(10)

where \( b^{vp} \) is a material parameter. This relation allows to rewrite (7):

\[ \Phi = \Phi_{vp} + \Phi_d \geq 0 \]

\[ \Phi_d = Y_d \dot{D} \]  

\[ \Phi_{vp} = \sigma^{vp} : \dot{e}^{vp} - \alpha_{vp} \dot{\gamma}_{vp} \]

(11)

where \( \sigma^{vp} = \sigma + b^{vp} \pi_{eq} \delta \) is the plastic effective stress (Coussy, 2004), \( \delta \) the second order identity tensor, \( \Phi_{vp} \) and \( \Phi_d \) the dissipations associated with viscoplastic deformation and damage, respectively. In the sequel, we seek to satisfy (11) by the sufficient conditions: \( \Phi_{vp} \geq 0 \); \( \Phi_d \geq 0 \).

The first of these two inequalities suggests that irreversible evolution laws might be formulated in terms of the plastic effective stress which allows to ensure both simplicity and thermodynamic consistency in the context of phenomenological modelling.

It is well-known that non-associated flow rules, which require the knowledge of a viscoplastic potential \( G \), can describe more correctly the dilatancy effect in rocks (Cristescu, 1994). This potential should depend on the thermodynamic forces: \( (\sigma^{vp}, Y_d, \alpha_{vp}) \). For simplicity, we assume an additive decomposition of \( G \) into separate contributors, \( G_{vp}(\sigma^{vp}, \alpha_{vp}) \) for creep straining and \( G_d(Y_d) \) for damage, thus the evolution laws write:

\[ \dot{e}^{vp} = \dot{\lambda}^{vp} \frac{\partial G_{vp}}{\partial \sigma^{vp}} \]  

(12)

\[ \dot{\gamma}_{vp} = -\dot{\lambda}^{vp} \frac{\partial G_{vp}}{\partial \alpha_{vp}} \]  

(13)
\[ \dot{\mathbf{D}} = \dot{\lambda}^d \frac{\partial G_d}{\partial Y_d} \]  

(14)

where \( \dot{\lambda}^{vp} \geq 0 \) and \( \dot{\lambda}^d \geq 0 \) are the viscoplastic and damage multipliers, respectively.

The evolution laws are totally defined by means of the above multipliers and potentials. The relevant choices of these constitutive elements to satisfy both practical applicability and thermodynamic condition will be discussed hereafter.

3. DISCUSSION ON RELEVANT CHOICES OF CONSTITUTIVE ELEMENTS

As indicated above, the interfacial energy can be computed using the data of water retention curve. Let’s consider the elastic potential \( G_s \) which can be decoupled as follows:

\[ G_s \left( \mathbf{\varepsilon}^e, \pi_{eq}, D, \gamma_{vp} \right) = G_s^1 \left( \mathbf{\varepsilon}^e, \pi_{eq}, D \right) + G_s^2 \left( D, \gamma_{vp} \right) \]  

(15)

The last term \( G_s^2 \) (plastic part) is the energy stored in the system of dislocations of the solid skeleton and not dissipated as heat when creep occurs (Lemaitre & Chaboche, 1990), while the specific form of the elastic part \( G_s^1 \) can be suggested, inspired from (Bui, et al., 2016), as follows:

\[ G_s^1 = \frac{1}{2} \mathbf{\varepsilon}^e : \mathbb{C}(D) : \mathbf{\varepsilon}^e - \left[ b(D)\pi_{eq} - b_0\pi_{eq0} \right] \mathbf{\varepsilon}^e - \frac{1}{2} \beta(D)\pi_{eq}^2 + \sigma_0 : \mathbf{\varepsilon}^e - \phi_0\pi_{eq} \]

\[ + \beta_0\pi_{eq}\pi_{eq0} - \frac{1}{2} \beta_0\pi_{eq0}^2 \]  

(16)

where the subscript \( 0 \) stands for an initial value; \( \mathbf{\varepsilon}^e \) the elastic volumetric strain; \( \mathbb{C}(D), b(D) \) and \( \beta(D) \) the damaged stiffness tensor, Biot’s coefficient and pore compressibility, respectively.

Following (16), Equations (5) lead to:

\[ \mathbf{\sigma} - \sigma_0 = \mathbb{C}(D) : \mathbf{\varepsilon}^e - \left[ b(D)\pi_{eq} - b_0\pi_{eq0} \right] \mathbf{\delta}; \]

\[ \phi - \phi_0 = b(D)\mathbf{\varepsilon}^e + \beta(D)\pi_{eq} - \beta_0\pi_{eq0} \]  

(17)

Inspired from (Shao, 1998) (Shao, et al., 2006), the damage-dependent parameters write:

\[ \mathbb{C}(D) = 3K(D)\mathbf{J} + 2G(D)\mathbb{K}; \quad \mathbf{J} = \frac{1}{3} \mathbf{\delta} \otimes \mathbf{\delta}; \quad \mathbb{K} = \mathbb{I} - \mathbf{J} \]

\[ K(D) = K_0 \left( 1 - a_1D \right); \quad G(D) = G_0 \left( 1 - a_2D \right) \]

\[ b(D) = 1 - \frac{K(D)}{K_s}; \quad \beta(D) = \frac{b(D) - \phi_0}{K_s} \]  

(18)
where \(\mathbb{I}\) denotes the fourth order identity tensor, \(K(D)\) and \(G(D)\) respectively the drained damaged bulk and shear moduli; \(K_s\) the bulk modulus of the solid phase; and \(0 \leq a_1, a_2 \leq 1\) are two parameters, determining the degradation of elastic moduli.

Then, using (8), (15) and (16), \(Y_d\) can be explicitly calculated:

\[
Y_d = \frac{1}{2} K_0 a_1 \left( e^e + \frac{\pi_{eq}}{K_s} \right)^2 + G_0 a_2 e^e : e^e - \frac{\partial G_s^2}{\partial D}
\]

Microcracking reduces the strain energy stored in dislocations, thus \(G_s^2\) should be decreasing with damage and consequently \(Y_d \geq 0\). For any evolution law satisfying the trivial condition \(\dot{D} \geq 0\) (since if healing is not considered here), the following inequality holds:

\[
\Phi_d = Y_d \dot{D} \geq 0
\]

In other words, to satisfy the thermodynamic consistency, the damage evolution law is not necessarily a function of \(Y_d\), a variable which is complicated to evaluate (see (19)). This provides a great freedom of postulating pragmatic and physically sensible damage evolution laws while always satisfying the thermodynamic constraints.

Finally, inspired from (Perzyna, 1966), the viscoplastic multiplier writes:

\[
\dot{\lambda}^{vp} = \frac{1}{K_{vp}} \langle \Phi(F_{vp}, y_{vp}, D) \rangle
\]

where \(K_{vp}\) denotes the viscosity, \(\langle ... \rangle\) the Macaulay’s brackets \((\langle x \rangle = (x + |x|)/2)\) and \(\Phi\) the overstress function. The yield function \(F_{vp}(\sigma^{vp}, \alpha_{vp})\) controls the overstress and implies that:

\[
\dot{\lambda}^{vp} = \begin{cases} 0 & \text{ if } F_{vp} \leq 0 \\ > 0 & \text{ otherwise} \end{cases}
\]

It remains to choose a potential \(G_{vp}\) that satisfies \(\Phi_{vp} \geq 0\) (condition \(\Phi_d \geq 0\) is separately fulfilled by (20)). It is proven that if \(G_{vp}\) is convex in the space of \((\sigma^{vp}, \alpha_{vp})\) and satisfies:

\[
G_{vp}(\sigma^{vp}, \alpha_{vp}) - G_{vp}(0, 0) \geq F_{vp}(\sigma^{vp}, \alpha_{vp}) \quad \forall(\sigma^{vp}, \alpha_{vp})
\]

then the non-negativity of \(\Phi_{vp}\) is a priori checked. Indeed, \(\Phi_{vp} = 0\) if \(F_{vp} \leq 0\), otherwise:

\[
\Phi_{vp} = \dot{\lambda}^{vp} \left( \sigma^{vp}; \frac{\partial G_{vp}}{\partial \sigma^{vp}} + \alpha_{vp} \frac{\partial G_{vp}}{\partial \alpha_{vp}} \right) \geq \dot{\lambda}^{vp} \left[ G_{vp}(\sigma^{vp}, \alpha_{vp}) - G_{vp}(0, 0) \right]
\]

\[
\geq \dot{\lambda}^{vp} F_{vp}(\sigma^{vp}, \alpha_{vp}) > 0
\]

If a model is built upon the above conditions, its thermodynamic consistency is automatically verified.

4. CASE STUDY: DEVELOPPING A MODEL FOR ROCKS

In the following, a new constitutive model will be constructed within the proposed framework to illustrate its applicability. The developed model aims at describing the effects of pore fluid pressures on the time-dependent behaviour of rocks. The Gibbs potential (((15) and
(16)) still hold. Inspired from (Ju, 1989), we choose $a_1 = 0$; $a_2 = 1$ in (18). Due to a lack of experimental data and as a first approximation, we assume that the solid matrix is viscoplastically incompressible, which is pertinent for brittle rocks in which creep is principally governed by sliding crack propagation (Coussy, 2004). A direct consequence of this hypothesis is $b^{vp} = 1$, which implies that the plastic effective stress reduces to $\sigma^{vp} = \sigma + \pi_{eq} \delta_s$, which coincides with the Terzaghi’s effective stress at full saturation.

The viscoplastic part is inspired from Lemaitre’s model, which is advantageous for the calibration of material parameters (Lemaitre & Chaboche, 1990). In practical terms, the overstress function is chosen as follows:

$$\Phi(F_{vp}, \gamma_{vp}, D) = \left( \frac{F_{vp}(\sigma^{vp}, \alpha_{vp})}{\sigma_r} \right)^N \frac{\gamma_{vp}^{-M}}{1 - D}$$

where $N$ and $M$ are two parameters controlling respectively the effect of stress and hardening on creep strain rate; $\sigma_r = 1$ MPa is a reference stress and $F_{vp}$ is the yield function defined by:

$$F_{vp}(\sigma^{vp}, \alpha_{vp}) = q + \eta_{vp} \sigma^{vp} - \alpha_{vp}$$

with $\sigma^{vp}$ the plastic effective mean stress, $q$ the Von-Mises deviatoric stress and $\eta_{vp} > 0$ a parameter accounting for the pressure-dependency of creep strain. The higher is the pore pressure, the greater is the overstress $F_{vp}$ and thereby, the higher is the viscoplastic strain rate. Concerning the viscoplastic potential, the following form is suggested:

$$G_{vp} = q + \eta_v \sigma^{vp} - \alpha_{vp}$$

On account of (26) and (27), the thermodynamic constraint (23) and hence, the condition $\Phi_{vp} \geq 0$, are satisfied if $\eta_{vp} \geq \eta_v$ is imposed. Since the model focuses on the pore pressure effect on the creep behaviour, we will assume, as a first approximation, that creep response is isochoric thereby $\eta_v = 0$, so that the thermodynamic constraint $\Phi_{vp} \geq 0$ is always satisfied in this case.

Note that isochoric creep strain has also been observed in many rocks (Arson, et al., 2012; Gasc-Barbier, et al., 2004). The convexity as well as the condition (23) is easily checked for $G_{vp}$, thus the non-negativity of $\Phi_{vp}$ is verified. Substituting (27) into (12) and (13) we obtain:

$$\dot{\epsilon}^{vp} = \dot{\lambda}^{vp} \frac{3s}{2q}; \quad \dot{\gamma}_{vp} = \dot{\lambda}^{vp}$$

These two equations in (28) lead to the consistent definition of the equivalent viscoplastic deviatoric strain currently encountered in literature:

$$\dot{\gamma}_{vp} = \sqrt{\frac{2}{3} \dot{\epsilon}^{vp} : \dot{\epsilon}^{vp}}$$

Hence, the hardening variable determined by the previous thermodynamic considerations identifies with viscoplastic distortion, which is a common choice in the literature (Shao, et al.,
Considering the lack of experimental data and for the sake of simplicity, we will assume $\alpha_{vp} = 0$ in the sequel. This is equivalent to the assumption that $G_S^*(D, \gamma_{vp})$ vanishes, which has also been considered in the original Lemaitre’s model.

To complete, inspired by Lemaitre’s model, the damage evolution law finally writes:

$$\dot{D} = \frac{1}{K_d} \left( \frac{F_d(\sigma^{vp})}{\sigma_r} \right)^{r_1} (1 - D)^{-r_2}$$

where $K_d$, $r_1$ and $r_2$ are parameters while $F_d(\sigma^{vp})$ is the damage criterion defined by:

$$F_d(\sigma^{vp}) = q + \eta_d \sigma^{vp}$$

where $\eta_d \geq 0$ is a parameter quantifying the pressure-dependency of the damage evolution. As $\dot{D} \geq 0$, the thermodynamic condition (20) is fulfilled.

The specific parameters of the model (viscoplastic and damage) can be identified with laboratory experiments, for instance from multi-step triaxial creep tests under drained condition. The parameter $M$ is classically determined by the same procedure as described in (Lemaitre & Chaboche, 1990). Then, integrating (28) on account of (25)-(27) and assuming that damage does not evolve during primary creep, we obtain for each experimental point “j”:

$$q_j = \left[ \frac{K_{vp}^{1/N}}{(M + 1)^{1/N}} \left( \frac{L_j}{(t_j - t_{ini})} \right)^{1/N} + \left( \frac{\eta_{vp} \sigma_{3j} + \pi_{eqj}}{3} \right) \right]$$

where $L_j = (|e_{1j}^{vp}|)^{M+1} - (|e_{1ini}^{vp}|)^{M+1}; q_j = \sigma_{3j} - \sigma_{1j}$ is the deviatoric stress; $t$ the time; and the subscript $ini$ denotes the initial value during each creep loading step. A nonlinear regression using functions of $y = ax^n + b$ type in the space of experimental points $\{L_j/(t_j - t_{ini}), q_j\}$ provides a first approximation of $N, \eta_{vp}$ and $K_{vp}$. The damage parameters can then be adjusted using the time-to-rupture value and the strain-time evolution during tertiary creep.

5. VALIDATION EXAMPLES BASED ON EXPERIMENTAL DATA

The proposed model is first applied to simulate the creep tests performed on saturated samples of a red sandstone located in the mining site of Chongqing, China (Xie, et al., 2016). The material parameters are summarised in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$E$</td>
<td>31000 MPa</td>
</tr>
<tr>
<td>$v$</td>
<td>0.15</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>0.6%</td>
</tr>
<tr>
<td>$b$</td>
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<tr>
<td>$\eta_{vp}$</td>
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</tr>
<tr>
<td>$K_{vp}$</td>
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</tr>
<tr>
<td>$N$</td>
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<tr>
<td>$M$</td>
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<tr>
<td>$\eta_d$</td>
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<tr>
<td>$K_d$</td>
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</tr>
<tr>
<td>$r_2$</td>
<td>0.5</td>
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Table 1. Model parameters for the Chongqing sandstone (Xie, et al., 2016)
Fig 1. Strain-time curves obtained from stepwise creep tests on Chongqing sandstone samples at confining pressure of 40 MPa and at pore pressure of: a) 5 MPa; b) 10 MPa; c) 15 MPa (circle dots represent experimental results reproduced from (Xie, et al., 2016)).

The experimental results and numerical simulations of three drained multi-step creep tests performed at different pore pressures (5, 10 and 15 MPa) are presented in Fig1 (a, b, c). The numerical results fit well with experimental data, showing the ability of the model to describe the effects of pore pressure and shear stress on the time-dependent deformation of this rock.

The model is then tested against a series of creep tests performed on unsaturated samples of a claystone located in Bure, France (Zhang & Rothfuchs, 2004). As failure is not observed, damage parameters cannot be determined for this series. The parameters are reported in Table 2.

\[
E = 4900 \text{ MPa}; \quad \nu = 0.3; \quad \phi_0 = 0.18; \quad b = 0.5; \quad n_{VG} = 1.9; \quad \alpha_{VG} = 0.068 \text{ MPa}^{-1} \\
\eta_{vp} = 0.095; \quad K_{vp} = 1.273 \times 10^{12} \text{ s}; \quad N = 3; \quad M = 1.8;
\]

Table 2. Model parameters for the Bure claystone (Zhang & Rothfuchs, 2004)

Note that \(n_{VG}\) and \(\alpha_{VG}\) are Van-Genuchten’s water retention parameters (Van Genuchten, 1980). Fig 2 shows the results of a uniaxial undrained stepwise creep test. The initially saturated sample is subjected to four creep loading steps (S1-S4) before being exposed to a drying process (S5) under an imposed relative humidity RH\(\approx\)24%. The sample is then submitted to a creep stage (S6) after drying. The numerical results fit well with the experimental data (Fig5a), for both the creep and drying phases. At the final step (S6), the sample being unsaturated, creep strain becomes very slow, showing that not only strain hardening but also suction opposes delayed deformations.
Fig 2. Undrained uniaxial creep test on an initially saturated argillite sample subjected to creep loading (S1-S4; S6) and drying (S5) (circle dots represent experimental results reproduced from (Zhang & Rothfuchs, 2004)

6. CONCLUSIONS

In this paper, a thermodynamic framework for the modelling of the time-dependent behaviour of unsaturated rocks is proposed. Viscoplasticity and isotropic creep damage are accounted for using the effective stress concept derived from the theory of poromechanics. The framework is robust for formulating simple constitutive models capable to reproduce the main features of the delayed deformation of rocks, while satisfying rigorous thermodynamic constraints. The applicability of this framework is demonstrated by formulating a specific model, of which the consistency and validity are confirmed by comparing experimental data and numerical simulations for different materials (sandstone, claystone) under both saturated and unsaturated conditions following different loading paths.

7. REFERENCES


